WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS METHODS
SEMESTER ONE 2018 TEST 3
DIFFERENTIAL CALCULUS APPLICATIONS, DISCRETE RANDOM VARIABLES, BERNOULLI TRIALS AND BINOMIAL DISTRIBUTIONS

Thursday 12th April

Name: _____

Time: 50 minutes **Part A:** ___/20 **Part B:** ___/30 **Total:** ___/50 _____ **%**

- Answer all questions neatly in the spaces provided. **Show all working.**
- You are permitted to use the Formula Sheet for both sections, and an A4 page of notes, plus up to 3 permitted calculators in the Calculator Allowed section.

Topic	Confidence
Further differentiations and applications • The second derivative and applications of differentiation	Low Moderate High
Discrete random variables • General discrete random variables	Low Moderate High
• Bernoulli distributions	Low Moderate High
• Binomial distributions	Low Moderate High

Self reflection (eg. comparison to target, content gaps, study and work habits etc)

Calculator Free Section **20 minutes** **/20**

1. [8 marks]

The displacement, x cm, of a particle at time t seconds, moving along a horizontal track is described by the function $x = 5\cos(3t)$.

a) Determine the initial position and velocity of the particle.

[3]

b) Determine the exact time when the particle first turns around.

[2]

c) Determine the exact rate of change of speed of the particle when $t = \frac{\pi}{4}$ seconds.

[3]

2. [7 marks]

Jack was investigating the variance of binomial distributions for different probabilities and exploring the connection to calculus.

a) For a random variable Y, where $Y \sim \text{Bin}(5, 0.4)$, calculate the variance, $\text{Var}(Y)$.

[2]

b) For the general random variable X, where $X \sim \text{Bin}(n, p)$,

i) Determine a function in terms of the probability p, for the variance, $\text{Var}(X)$.

ii) Use calculus techniques to show that the maximum variance is achieved when $p = 0.5$. Justify that your result is a maximum.

[5]

3. [5 marks]

A discrete random variable X has the following properties:

- the expected value $E(X) = 18$
- the standard deviation $\sigma = \frac{3\sqrt{5}}{2}$.

a) If the random variable is binomial, determine the number of trials and probability of success.

[3]

b) Determine the expected value $E(Y)$ and variance $\mathrm{Var}(Y)$ if Y is a random variable such that $Y = 5 - 2X$.

[2]

Name: _____

Calculator Allowed Section **30 minutes** **/30**

4. [11 marks]

Consider the function $y = \frac{10\ln(x)}{x^2}$.

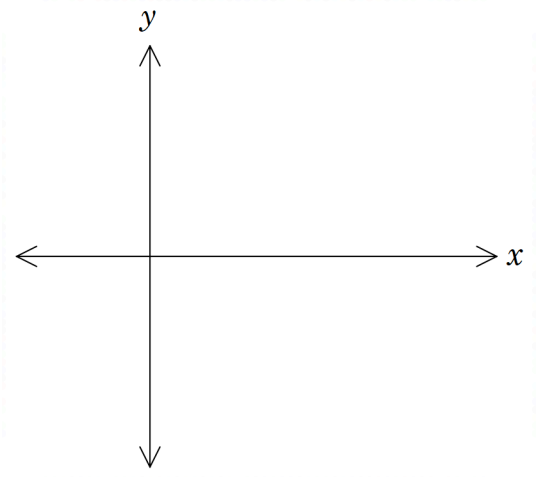
a) Determine $\frac{dy}{dx}$ and its associated domain. Hence determine the exact location and nature of the stationary point(s).

[5]

b) Determine the exact location of any inflection points.

[3]

c) Sketch the graph of the function labelling key features (to 2 decimal places).



[3]

5. [9 marks]

Aaron and Brad are playing a tennis match. The match continues until one player wins a total of two (2) sets. Aaron estimates from past experience that his chance of winning any set against Brad, independent from any previous sets, is $\frac{3}{10}$.

Let the random variable X be the number of sets won by Aaron in the match.

a) Give a reason as to why X cannot be modelled by a binomial distribution.

[1]

b) Draw a tree diagram to show the possible outcomes of the match and the associated probabilities. Hence complete the probability density function for X in the table below, stating answers as fractions.

x	0	1	2
$P(X = x)$			

[4]

c) Determine the probability that Aaron wins the match, given he wins the first set.

[2]

d) Calculate the expected value of X as a decimal, and explain its meaning in the context of the question.

[2]

6. [7 marks]

Based on shipments of mobile phones to Australia in the last quarter of 2017, the Apple iPhone has a market share of around 37%[i]. Assume that every Australian has exactly one mobile phone.

A random survey of 20 people was conducted on mobile phone type. Showing appropriate probability notation, determine the probability, to three decimal places, that

a) Exactly six respondents had an iPhone.

[2]

b) At least six respondents had an iPhone.

[2]

c) No more than ten respondents had an iPhone, if it is known at least six had an iPhone.

[3]

7. [3 marks]

How many times should a fair die be rolled so that the probability of rolling exactly one six is the same as the probability of not rolling a six at all?

[3]

[i] https://www.statista.com/statistics/436033/australia-smartphone-shipments-vendor-market-share/